Mathematics Across Cultures and Disciplines

By Dr. Abdulkeni Zekeria.

Introduction

Although some people study mathematics for enjoyment, A majority of students of mathematics ask at some point in their study…" when will I ever need this? "

Many people think mathematics has nothing to do with them. It has to do with numbers and maybe some *x*'s and *y*'s which have no practical use.

This pessimistic outlook creates a negative altitude in students which is a problem for teacher of mathematics. In this paper I will summarize the historical development of mathematics and the contributions from different cultures as well as some of the applications of mathematics in different areas in our daily lives. I hoped that this will help students to appreciate mathematics. In section one, I will cover briefly the history of mathematics in recorded history, and in section two some application of mathematics will be considered

Section One

Different civilizations have contributed to the body of knowledge mathematics. Different cultures have always had different ways of solving their practical problems. The more civilized and advanced a culture is, the more they develop mathematics. Progress in mathematics and science is hampered by socioeconomic constraints. During the last three hundred years, there was tremendous progress in development of mathematics and sciences in Europe unparalleled in human history, which enabled the Europeans to dominate the world.

Initial development of our mathematics can be traced to the Babylonians, Egyptians, Greeks, Chinese, Indians, Muslims and Europeans. Among, the great rivers of Africa and Asia, new forms of society, appeared. The Nile in Africa, the Tigris and Euphrates in western Asia, the Hwang Ho and the Yangtze in eastern Asia, Indus and the Ganges in south-central Asia all gave rise to civilizations. It was necessary to have knowledge of flood control, irrigation, convenient transportation, projects of engineering, and financing to do business. To have these technical skills it was necessarily to develop mathematics. The effort to systemize the concept of size, shape, numbers and counting is regarded the earliest form of mathematics. To pursue practical sciences in agriculture, engineering, and business, required computation of usable calendar. The development of weights and measurements were needed to serve harvesting and trade. Most societies used deductive methods for solving problems. This approach was an example of "do thus and so "method in order to solve a problem or perform an operation, a cookbook-like recipe was given and was performed over and over to solve a similar problem. Tendencies toward abstraction were bound to develop, and some extend the science, the science was then studied for its own sake. It was in this way that algebra untimely evolved from arithmetic and the beginning of theoretical geometry grew out of measurement. The classical Greek period gave rise to more formal

types of mathematics in which general concepts were applied to specific problems resulting in the structural, logical development of mathematics.

1. Babylon

Along the banks of Tigris and Euphrates rivers in western Asia, the Babylonian civilization developed. The Babylonians used wedge-shaped cuneiform script for writing on clay tablets; tablets were inscribed whilst the clay was moist, and baked in an oven by the heat of the sun. These clay tablets remain intact today. They did not perish. Now there are approximately half-million tablets, about 400 have been identified strictly mathematical tablets a good half of them contain mathematical tables.
Babylonian geometry is intimately related to practical measurements. They must have been familiar with the general rules for the area of rectangle, areas of right and isosceles triangles, trapezoid, volume of rectangular parallelepiped.
By 2000 B.C. Babylonian arithmetic had evolved into a well developed rhetorical, or prose algebra.
By studying the tablets such as Plimpton 322(in Colombia University) mathematicians are discovering that ancient Babylonians were indefatigable table makers, computers of high skill and definitely stronger in algebra than geometry. Babylonian discussion of some quadratic, cubic, and biquadrate equations; interesting approximations of $\sqrt{2}$, $1/\sqrt{2}$, and π are found in their tables.

2. Egypt

In Egypt, the Great Pyramid of Giza was build with knowledge of mathematics and engineering. Egyptians used stone and papyrus for writing which still remain today. Our knowledge of their mathematics comes from Moscow papyrus that contains 25 problems, and the Rhind papyrus with 85 problems. Many of the 110 problems in the Rhind and Moscow papyri show their practical origin. Although most of the problems have practical origin, there are some with more theoretical nature.
26 of the problem are geometric many stem from mensuration formulas needed for computing land areas and granary volume. The area of a circle is taken as equal to that of the square on 8/9 of the diameter and the volume of a right cylinder as the product of the area of the base by the length of the altitude.
The Egyptians have interesting methods of multiplication and division. Multiplication and division were usually performed by a succession of doublings and use of unit fraction. The method of false position was used to solve what we call today “word problems”. There is evidence that other mathematical knowledge of Egyptians including: prime numbers, composite numbers, arithmetic means, geometric means, arithmetic and geometric series.
Finally, Egyptians have some form of symbolism of algebra, and they have a method for solving linear and certain types of quadratic equations.

3. Greek

The great scientists of the ancient world lived in tiny Greece. The two main city states were Sparta and Athens. Sparta was strong in military and Athens was a commercial center. Despite the political disunity, chronic food shortage, overpopulation and almost constant warfare, the Hellenic Age in Greece witnessed remarkable intellectual achievements. In the Agoras, market place of Athens and other city-states, philosophers taught students logic, poetry, mathematic, philosophy. It was the application of deductive reasoning to mathematics by Thales of Miletus and Pythagoras, and the foundation of modern medicine by Hippocrates of Cos more than 2000 years ago, that laid the foundation of western civilization.

Men began to ask why as well as how things worked. Applying deductive reasoning such as "Why are the base angles of an isosceles triangle equal?" , "why does the diameter of a circle bisect the circle?"," Why are the vertical angles formed by two intersecting lines are equal?","Why are two triangles congruent if they have two angles and one side in each respectively equal?" and " Why angle inscribed in a semicircle is a right angle?" they came with correct conclusions. Thales of Miletus is credited with the elementary geometry results of the above questions. The results are taught in the high school geometry of today.

The Pythagorean Theorem is, among, the great achievement of the Greek mathematics. Other great development in mathematics by the Greeks include: the discovery of irrational numbers, the geometric solution of quadratic equations, transformation of area , the Regular Solids, and the three famous problems: The duplication of cube, the trisection of an angle, and he quadrature of the circle.
The three centuries of Greek mathematics, commencing with efforts of demonstrative geometry by Thales about 600 B.C and culminating with remarkable Elements of Euclid about 300 B.C. constitute a period of extraordinary achievement.

4. China

The Chinese and Indians used perishable media like bark and bamboo for their writing so very little is known about their earlier work in mathematics.
After the decline of classical Greek mathematics, the mathematics of China became one of the most prosperous in the world. Printing originated in 8th century. The first mathematics book of china was printed in 1084. During the Sung and Yuan dynasties in the 10^{th} - 14^{th} century many mathematicians flourished and many books appeared. Among the major mathematicians were Ch'in Kiushao, Li Yeh, Yang Hui and the greatest of all Chu Shiikie.
Some of their achievements are: the creation of a positional decimal numerical system, the acknowledgement of negative numbers, a more précis value for π, arrival at Horner's method for numerical solution of algebraic equation, solution of system of linear equations by matrix, the awareness of binomial theorem, solution of system of simultaneous congruencies by so-the called Chinese Remainder Theorem, the development of decimal fractions, the development of rule of three, the application of rule of double false position , the development of arithmetic series of higher order, and development of descriptive geometry. Many of the Chinese findings in math ultimately made their way to Europe via India and Arabia.

5. Indian Subcontinent

Little is known of the development of mathematics of ancient Indian subcontinent. The ruins of the city at Mohwnjo Daro northwest of Karachi are 5000 year old. Here there is evidence of wide streets, brick dwelling, tiled bathrooms, covered city drain; community swimming polls indicate high civilization. These early people had a system of writing, counting, weighing, and measuring. They dug canals for irrigation- all these required basic mathematics and engineering.

The most famous ruler of The Maurya Empire was King Asoka (272-232 B.C.).He has erected stone pillars in every important city of his day. Some of the pillars are still standing, and contain the earlier preserved specimens of our present number symbols. After Asoka, there were many invasions, until the coming of Indian emperors of the Gupta dynasty. The Gupta period (about 400 A.D.) was the golden age of the Sanskrit renaissance and India became the center of learning, art and medicine.

The prominent mathematicians of India are:

Aryabhatas: wrote work on astronomy in the 6th century

Brahmagupta: who has worked in the astronomical center of Ujjain in central India in the 7th century

Mahavira: who was from Mysore in southern India wrote a book on elementary mathematics in 850 A.D

Bhaskara:who wrote arithmetic and algebra books. There was little progress after him until modern day.

The most famous Indian mathematician of all times is Srinivvas Ramanujan (1887- 1920). The publication of Ramanujan's notebooks in 1920, has disclosed many facets of the man's unusual genius.

Indian contribution to mathematics include: the present positional numerical system, the development of algorithm for arithmetic operations, the solution by false position, the solution commercial problems, the summations arithmetic and geometric progressions, admission of negative numbers and irrational numbers, the solution of quadratic equations by the method of completing square (Hindu method). Their geometry was largely empirical and connected with mensuration. They used Pythagorean relations in the construction of altars. They gave Herons formula for triangles and gave an extension for a cyclic quadrilateral having sides a, b, c, and d. They also gave more accurate value for π.

Like the Greeks, Indians regarded trigonometry as a tool for their astronomy. They used the familiar degrees, minutes, seconds to construct tables of sines.

Indian mathematics is very uneven in quality, its good and poor mathematics often appearing side by side. Al Biruni put in a well known book, that India in contrast to the uniformity high quality of Greek mathematics; "Hindu mathematics was a mixture of pearl shell and sour date… of costly crystal and common pebbles."

6. Muslim

During the peak of Muslim civilization , the Abbasids Caliphs (Kelifa Al-Mansur, Kelifa Harun Al-Rahid ,and Kelifa Al- Manun), in the 8^{th} century, Baghdad was made the center of learning of the world, by building an observatory in Baghdad, inviting scholars from east and west, and encouraging scholarships. During this period classic works of Greek and India were translated into Arabic. Brahhmgupta's works

were translated (around 766A.D.). Hindu numerals were turned into Muslim Mathematics. Likewise several Greek classics were translated into Arabic, among them Euclid's "Elements".

Prominent mathematicians in the Islamic era.

Mohammed ibn Musa Al-Khwarizmi: who was from Persia, wrote a treaty on algebra and a book on Hindu numerals which later influenced Europeans mathematics when translated into Latin in the 12^{th} century.

Thabit ibn Qorra: who was from Haran, a famed as physician, philosopher, linguist and mathematician made most satisfying translation of the "Elements ", in the 9^{th} century.

Abu'l-Wefa : who was from Khorrasan was the most celebrated Muslim mathematician. He translated the works Diophantus into Arabic, introduced tangent function into trigonometry and computed tables for sine and tangent for interval of 15'.

Al-Haitam: who was from Basra, wrote in a number of mathematical topics. He was a physicist who is famous for his work on optics.

Abu Kamil: wrote commentary on Al-Khwarizmi which was drawn by Fibonacci 12^{th} A.D. al-Karkhi –and 11^{th} century worked into algebra

Omar Khayyam: who was from Khorasan gave geometrical solution of cubic equations. He is also noted for his accurate proposed calendar reform, and his work on plane and spherical trigonometry independent of astronomy. Has discussed the flaws of parallel postulate thus he laid foundation for non-Euclidean geometry. He is also credit for the original proof of the Pythagorean Theorem

Nasir ed-din Altusi: who was from Khorasan wrote on astronomy, biology, chemistry and mathematic. He was a prolific writer.

Ulugh Beg (15th century) of Persian astrometry compile remarkable tables of sines and tangents for 1' interval correct to eighth or more decimal places.

Al-Kashi gave more accurate approximation of π, important work on decimal fraction and dealt with binomial theorem in the "Pascal triangle" form.

Like the Hindus the Muslim mathematicians generally regarded themselves as primarily astronomers and thus showed considerable interest in trigonometry. They may also be credited with using all the six trigonometric function and with improvements upon derivation of the formulas of spherical trigonometry.

There is not an agreement of the contributions of the Muslims to the mathematics. Some have assigned high marks for originality and genius of the Muslim mathematicians, in particular in algebra and trigonometry, while others think that, while they are learned scarcely creative, and their work is secondary, both in quantity and quality. There is an agreement however, that the Muslim mathematicians served admirably as custodian of much of the worlds intellectual possessions which were later transmitted to Europeans after the European Dark Ages.

7. European

After the fall of Roman Empire little mathematical development was achieved in Europe.

Fibonnaci was the greatest mathematician of Middle Age Europe (476- 1492 A.D.)
He translated the work of Al-Khwarizmi. In his book Libre *abbaci* (1202) he advocated the replacing of the Roman Numerals by Hindu –Arabic Numerals. Other books of his treated topics in geometry, algebra and number theory.

Dawn of modern mathematics: 15 – 17th century

The 15th century was the beginning of algebraic symbolism and solutions of some cubic and quadratic equations. Prominent mathematicians includes: Cardino , Tartaglia, and Viete.

The explosion of mathematics after the16th century.

After the European renaissance, the development of mathematics in Europe is unsurpassed both in quality and quantity. Most of the breakthroughs in mathematics took after 16th century.
The contribution of Europeans to mathematics in the last four centuries is so; important and huge it is difficult to list them. I will attempt to list sample mathematicians and branchs of mathematics. Many of them are familiar to any student who took calculus.
Napier, Descartes, Fermat treated logarithms, analytic geometry, number theory respectively.
Newton and Leibniz are credited for invention of calculus in 17th century.
The Bernoulli Family, Taylor and Maclaurin contributed to the advancement of calculus.
Cauchy and Reimann contributed to complex analysis.
De Moivre, Laplace, and Legendre contributed to theory of probability.
Euler is credited has work on topology and other mathematics.
Gauss- is the most prolific of all and is considerd the prince of mathematics.
Abel, Galois, and Lagrange contributed to abstract algebra
Lobachevskian contributed to non- Euclidian Geometry
Boole, and De Morgan treated logic and algebra

8. Others

Other civilizations and cultures have also contributed to the development of mathematics.
They may use different symbols and notations. For instance, the civilizations in central and South America was relatively developed before coming of the Europeans. They must have some development of mathematics to achieve the kind of development they had.

Section Two

In addition to the cultural connections to the development of mathematics, teachers must be aware of the beauty and application of mathematics in different disciplines so that the enthusiasm will be transmitted to students.
The current research and investigation of mathematics have practical application as their origins as well as theoretical area that might have started initially on solving practical problems. Examples where mathematics is applied include: management sciences, social sciences, statistics, arts, biological sciences and geography.

1. Management

Management Sciences uses tools from mathematics to solve management problems. Tools such as linear programming, graph theory, and graph theory concept of trees, are all used in solving problems in urban services, business efficiency, schedule and planning, and linear programming.

Despite the complications many *real –world* problems such as providing services to different part of a city, planning for disaster recovery, the Apollo Project (missions to the moon), can be solved by using operations research.

Historically, the question of running business efficiently was answered by trial and error methods, but now operations-research techniques exist to minimize cost while doing all tasks in the shortest possible time.

The issue of common a Ancestors can be looked at from the standpoint of view of the reconstruction of ancient manuscript from the remaining few original manuscript such as the Euclid's *Elements* and Chaucer's *Canterbury Tale,* Similarly psychologists may be interested in which colors people perceive as being closely related and compare those of people who are color blind. Biologists are interested in determining which species are more closely related to each other including pieces known only in fossil form. All this can be studied using graph theories, specifically the graph theory concept of a tree. With discovery of molecular biology, many new avenues have been opened up. We can now draw trees of relatedness based on an organisms' genetic material, DNA, or the proteins that the DNA codes for.

Linear programming has originated in WW II first for military purposes, but now they are used, in much aspect of our lives such as industry, schools, local governments.

2. Social Science

As in the 16^{th} century the invention of calculus accelerated the progress of natural sciences which was needed for the industrial revolution, today there is revolution of sort in the social sciences. The application of mathematics to the study of human beings- their behavior, values, interaction, conflicts and the methods of making decision. Decision making is being influenced profoundly by modern mathematics. Several specialized mathematics areas had created primary to assist in arriving at sound decisions.

How does a group, each member with his or her set of values, select one outcome from a list of possibilities?

May's Theorem says if the number of voters is an odd number and there are only two candidates then the majority rule is the best system in satisfying desirable conditions (voters are treated equally, candidates are treated equally...). If the number of alternatives is more than two, then we have what is known as **Arrow's impossibility theorem** which says that it is impossible to find a voting system that satisfy certain desirable conditions .

3. Statistics

Is there genetic difference between two related types of cancer? To find out scientists use "microarrays" to report the activities of thousands genes at once. Checkout scanners and microarrays produce immense amounts of data, numerical facts. So do opinion polls, medical studies and even sport pages.

We learn from data by making graphs, making calculation and analyzing data. *Statistics* is the sciences of collecting, organizing, and interpreting data.

4. Arts

Mathematicians search for and classify numerical, geometrical and even abstract patterns. Numerical patters include the Golden Ration, the Fibonacci numbers. Geometrical pattern as such as the beautiful nautilus grow according to very strict specific spiral pattern. Botanist have appreciated other spirals such the pinecone and the sunflower. "The sense delight in things duly proportional"So said Thomas Aquinas in noting human aesthetic appreciation, particular symmetry. Balance in symmetry patterns obtained by rigid motion, such as reflection, rotation, translation and glide reflection preserve the size and the shape of figures. Patterns Created by the Bakuba People of Republic of Congo uses strip patterns. Analyze on ancient pottery and suggest new and beautiful artistic designs.
People cover floors and the walls of their houses and public place places of worship by selecting shapes and colors to form pleasing designs. The same intricacy and complexity arise in other decorative arts on carpets, fabrics, baskets, and. Tessellation or tiling is the use of repeated shapes to cover a flat surface without gap or overlaps. The major mathematical question about tiling is: given one or more shapes of tiles, can they tile the plane? If so, how?
M.C. Escher (1898- 1972) was inspired by the decorations in tiles in the Alhambra in Spain. He spent much of his career of making prints to create tiling using shape of living beings.

5. Biology

In biology, exponential functions are used to find the exponential growth of bacteria. While carbon dating and radioactivity uses exponentially decaying substance. The same mathematical models are applicable to other areas of life. Consumer financial models are often versatile and flexible. Growth of money is like the growth of some biological population. Inflation of a currency or depreciation of an asset is like the decay of a radioactive substance. Finding out how long a retirement "nest egg" will last is similar to determining how long before a nonrenewable resources, such as oil or coal, may be exhausted. Managing a trust fund, such as the endowment of a college, presents problems similar to the management of a renewable biological resource such as a forest or a fishery. In an Islamic financing, the fixed interest charged by lending institution is replaced by sharing risk taking returns. The growing Islamic financing market attempts to eliminate usury in the agreements otherwise uses the same model.

6. Geography

In geography one may want to know the direction form one point to another point on the globe. Euclidean geometry is sufficient to solve the problem of direction if we assume that the Earth flat.

Non-Euclidian geometry has to be used if the assumption that the earth is flat removed.
It took few years for Muslim in North America to correct the direction of Qibla from southeast to the correct direction of roughly northeast- the direction of the greater circle- the direction of the plane through two points on the globe (say any place in US and Mecca) and the center of earth. The incorrect direction was based on a relative flat map where the Americas are placed of the left side of the map

Modern mathematics – differential geometry is the tool to be used in determining the Qibla (direction of Mecca) from any place on earth.

Conclusion

Muslim teachers should be aware of the contributions made to mathematics by Muslim scientists, as well as by other civilizations. Application to other discipline should be emphasized. Many mathematics textbooks contain historical annotates on the margins or at the end of a chapter.

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